Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WMA14/01**

Mathematics

International Advanced Level
Pure Mathematics P4

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 11 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

P 7 2 0 6 8 A 0 1 3 2

Pearson

1. A curve C has parametric equations

$$x = \frac{t}{t-3} \qquad y = \frac{1}{t} + 2 \qquad t \in \mathbb{R} \qquad t > 3$$

Show that all points on C lie on the curve with Cartesian equation

$$y = \frac{ax-1}{bx}$$

where a and b are constants to be found.

(3)

Rearrange to make t the subject:

$$x = \frac{t}{t-3}$$

$$xt - 3x = t$$

$$xt - t = 3x$$

$$t(x-1) = 3x$$

$$t = \frac{3x}{x-1} \quad \text{M1}$$

Substitute into $y = \ldots$:

$$y = \frac{x-1}{3x} + 2 \quad \text{dM1}$$

$$y = \frac{x-1}{3x} + \frac{6x}{3x}$$ get common denominator

$$y = \frac{7x-1}{3x} \quad \text{A1}$$

P 7 2 0 6 8 A 0 2 3 2

Question 1 continued

(Total for Question 1 is 3 marks)

2. (a) Express $\dfrac{3x}{(2x-1)(x-2)}$ in partial fraction form.

(3)

(b) Hence show that

$$\int_5^{25} \frac{3x}{(2x-1)(x-2)}\,dx = \ln k$$

where k is a fully simplified fraction to be found.

(Solutions relying entirely on calculator technology are not acceptable.)

(4)

(a) We are asked to do Partial Fractions

$$\frac{3x}{(x-2)(2x-1)} \equiv \frac{A}{2x-1} + \frac{B}{x-2} \quad \text{M1}$$

Manipulate this to get common denominator

$$\frac{3x}{(x-2)(2x-1)} \equiv \frac{A(x-2)+B(2x-1)}{(x-2)(2x-1)}$$

Equate the numerators and substitute in values in order to get A, B

$$3x \equiv A(x-2) + B(2x-1)$$

choose values to substitute in so that brackets become 0, to make your life easier!

$x=2$: $3(2) \equiv A(\underbrace{2-2}_{0}) + B(4-1)$

$6 = 3B \Rightarrow \boxed{B=2}$

$x=\frac{1}{2}$: $\frac{3}{2} \equiv A\left(-\frac{3}{2}\right) + B(\underbrace{1-1}_{0})$

$3 = -3A \Rightarrow \boxed{A=-1}$ A1

$$\therefore \frac{3x}{(x-2)(2x-1)} = \frac{2}{x-2} - \frac{1}{2x-1} \quad \text{A1}$$

P 7 2 0 6 8 A 0 4 3 2

Question 2 continued

(b) $\int_5^{25} \frac{3x}{(x-2)(2x-1)}\,dx = \int_5^{25} \frac{2}{x-2} - \frac{1}{2x-1}\,dx$ P.F. from (a)

$= \left[2\ln(x-2) - \frac{1}{2}\ln(2x-1)\right]_5^{25}$ M1A1

Reverse chain rule: divide by the derivative of the Bracket.

$= \left(2\ln 23 - \frac{1}{2}\ln 49\right) - \left(2\ln 3 - \frac{1}{2}\ln 9\right)$ dM1

$= (2\ln 23 - 2\ln 3) + \left(\frac{1}{2}\ln 9 - \frac{1}{2}\ln 49\right)$ group the coefficients together and apply ln rule: $\ln a - \ln b = \ln\frac{a}{b}$

$= 2\ln\frac{23}{3} + \frac{1}{2}\ln\frac{9}{49}$ apply ln rule: $a\ln b = \ln b^a$

$= \ln\left(\frac{23}{3}\right)^2 + \ln\left(\sqrt{\frac{9}{49}}\right)$

$= \ln\frac{529}{9} + \ln\frac{3}{7}$ apply ln rule: $\ln a + \ln b = \ln a.b$

$= \ln\frac{529}{\cancel{9}3} \cdot \frac{\cancel{3}1}{7}$

$= \ln\left(\frac{529}{21}\right)$ A1

(Total for Question 2 is 7 marks)

3.

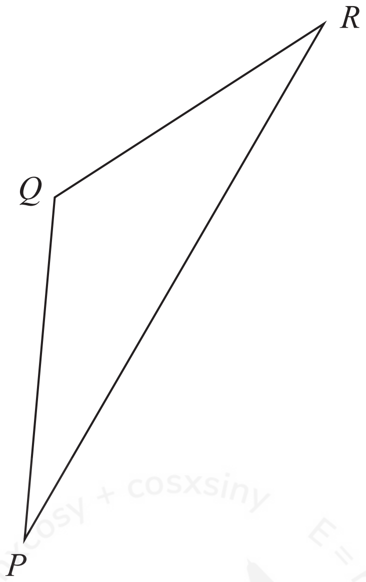


Figure 1

Figure 1 shows a sketch of triangle PQR.

Given that

- $\overrightarrow{PQ} = 2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$
- $\overrightarrow{PR} = 8\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}$

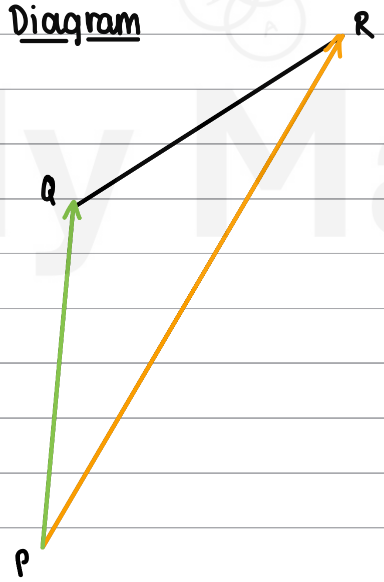
(a) Find $\overrightarrow{RQ}$

(2)

(b) Find the size of angle PQR, in degrees, to three significant figures.

(3)

(a) Diagram



$\overrightarrow{RQ} = -\overrightarrow{PR} + \overrightarrow{PQ}$

$= -\begin{pmatrix} 8 \\ -5 \\ 3 \end{pmatrix} + \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}$ M1

$\overrightarrow{RQ} = \begin{pmatrix} -6 \\ 2 \\ 1 \end{pmatrix}$ A1

Question 3 continued

(b) Use Formula:

$\cos\theta = \frac{\bar{a} \cdot \bar{b}}{|\bar{a}||\bar{b}|}$ Formula for dot product: $\begin{pmatrix} a \\ b \\ c \end{pmatrix} \cdot \begin{pmatrix} d \\ e \\ f \end{pmatrix} = ad + be + cf$

For $R\hat{Q}R$:

$$\cos(P\hat{Q}R) = \frac{\vec{RQ} \cdot \vec{PQ}}{|\vec{RQ}|\,|\vec{PQ}|}$$

$$= \frac{\begin{pmatrix} -6 \\ 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}}{\sqrt{6^2+2^2+1^2}\,\sqrt{2^2+3^2+4^2}}$$ M1 dM1

$$= \frac{-12 - 6 + 4}{\sqrt{41}\,\sqrt{29}}$$

$$\cos(P\hat{Q}R) = \frac{-14}{\sqrt{41}\,\sqrt{29}}$$

$R\hat{Q}R = 114^\circ$ to 3sf A1

(Total for Question 3 is 5 marks)

4.

$$g(x) = \frac{1}{\sqrt{4 - x^2}}$$

(a) Find, in ascending powers of x, the first four non-zero terms of the binomial expansion of $g(x)$. Give each coefficient in simplest form.

(5)

(b) State the range of values of x for which this expansion is valid.

(1)

(c) Use the expansion from part (a) to find a fully simplified rational approximation for $\sqrt{3}$

Show your working and make your method clear.

(2)

(a) Get the formula for the Binomial Expansion from the formula booklet:

$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots$, Range of validity $|x|<1$

We see that the given expansion is in form $(a+bx)^n$, we need to manipulate to get $(1+ax)^n$.

$$(4-x^2)^{-\frac{1}{2}} = (4)^{-\frac{1}{2}}\left(1-\frac{x^2}{4}\right)^{-\frac{1}{2}}$$
$$= \frac{1}{2}\left(1-\frac{x^2}{4}\right)^{-\frac{1}{2}}$$

B1

Substitute into the Formula:

$$\frac{1}{2}\left(1-\frac{x^2}{4}\right)^{-\frac{1}{2}} = \frac{1}{2}\left[1+\left(-\frac{1}{2}\right)\left(-\frac{x^2}{4}\right) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2!}\left(-\frac{x^2}{4}\right)^2 + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{3!}\left(-\frac{x^2}{4}\right)^3 + \ldots\right]$$ M1A1

$$= \frac{1}{2}\left(1 + \frac{x^2}{8} + \frac{3x^4}{128} + \frac{5x^6}{1024} + \ldots\right)$$

$$= \frac{1}{2} + \frac{x^2}{16} + \frac{3}{256}x^4 + \frac{5}{2048}x^6 + \ldots$$ A1A1

(b) Range of validity for $(1-x)^n \Rightarrow |x|<1$.

$\therefore\ 1 < 4 - x^2 < 1 \Rightarrow -2 < x < 2$

Hence $|x|<2$ is the Range of validity B1

(c) With $x=1$:

$$\frac{1}{\sqrt{4-(1)^2}} = \frac{1}{2} + \frac{1}{16}(1)^2 + \frac{3}{256}(1)^4 + \frac{5}{2048}(1)^6$$ M1

Manipulate the expansion to get $\sqrt{3}$ by taking the reciprocal.

$\therefore\ \frac{1}{\sqrt{3}} = \frac{1181}{2048}$, hence $\sqrt{3} = \frac{2048}{1181}$ A1

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Question 4 continued

(Total for Question 4 is 8 marks)

5. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

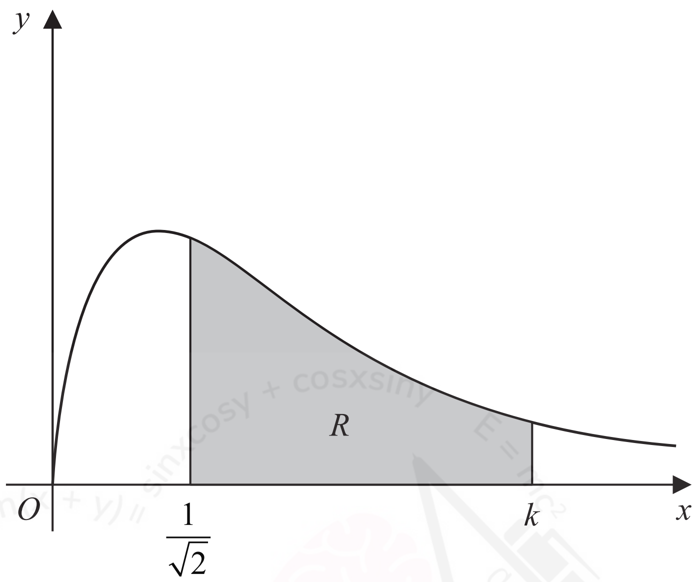

Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$y = \frac{12\sqrt{x}}{(2x^2 + 3)^{1.5}}$$

The region R, shown shaded in Figure 2, is bounded by the curve, the line with equation $x = \frac{1}{\sqrt{2}}$, the x-axis and the line with equation $x = k$.

This region is rotated through 360° about the x-axis to form a solid of revolution.

Given that the volume of this solid is $\frac{713}{648}\pi$, use algebraic integration to find the exact value of the constant k.

(6)

Question 5 continued

Formula for Volume of Revolution around the x-axis:

$$V = \pi\int_a^b y^2\,dx$$

where a and b are the points on the x-axis bounding the area

$$V=\pi\int_{\frac{1}{\sqrt{2}}}^{k}\left(\frac{12\sqrt{x}}{(2x^2+3)^{\frac{3}{2}}}\right)^2 dx$$ B1

$$=\pi\int_{\frac{1}{\sqrt{2}}}^{k}\frac{144x}{(2x^2+3)^3}\,dx$$ Apply the power

$$=144\pi\int_{\frac{1}{\sqrt{2}}}^{k}x(2x^2+3)^{-3}\,dx$$ Notice how this is in the form $\int f'(x)\cdot[f(x)]^n\,dx$.

Integration by Recognition

$\therefore = \frac{f(x)^{n+1}}{n+1}+c$

M1A1 $$=144\pi\left[\frac{(2x^2+3)^{-2}}{-2\cdot 4}\right]_{\frac{1}{\sqrt{2}}}^{k}$$

Reverse chain rule: divide by derivative of the Bracket

$$=\frac{144\pi}{-8}\left[(2x^2+3)^{-2}\right]_{\frac{1}{\sqrt{2}}}^{k}$$

given value for volume $\frac{713}{648}\pi = -18\pi\left[(2x^2+3)^{-2}\right]_{\frac{1}{\sqrt{2}}}^{k}$ M1

$$\frac{713}{648}\cancel{\pi}=-18\cancel{\pi}\left((2k^2+3)^{-2}-\left(2\left(\tfrac{1}{\sqrt{2}}\right)^2+3\right)^{-2}\right)$$

$$\frac{713}{648\cdot -18}=\frac{1}{(2k^2+3)^2}-\frac{1}{(1+3)^2}$$

$$-\frac{713}{11664}+\frac{1}{16}=\frac{1}{(2k^2+3)^2}$$

$$\frac{1}{729}=\frac{1}{(2k^2+3)^2}$$

$\therefore 729=(2k^2+3)^2$ ddM1

$27=2k^2+3$

$24=2k^2$

$12=k^2$

A1 $k=2\sqrt{3}$ (no other solution since k is +ive)

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Question 5 continued

Question 5 continued

(Total for Question 5 is 6 marks)

6.

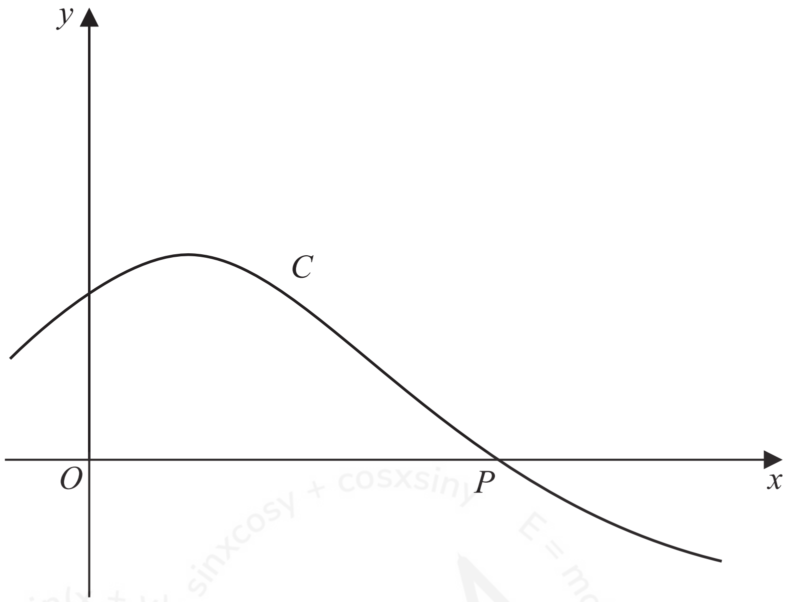

Figure 3

Figure 3 shows a sketch of the curve C with parametric equations

$$x = 1 + 3\tan t \qquad y = 2\cos 2t \qquad -\frac{\pi}{6} \leqslant t \leqslant \frac{\pi}{3}$$

The curve crosses the x-axis at point P, as shown in Figure 3.

(a) Find the equation of the tangent to C at P, writing your answer in the form $y = mx + c$, where m and c are constants to be found.

(5)

The curve C has equation $y = \mathrm{f}(x)$, where f is a function with domain $\left[k, 1 + 3\sqrt{3}\right]$

(b) Find the exact value of the constant k.

(1)

(c) Find the range of f.

(2)

Question 6 continued

(a) At P, $y=0$. Use this to get t at P:

$$0 = 2\cos 2t$$
$$0 = \cos 2t$$
$$t = \frac{\pi}{4}.$$

Hence P_x:

$$x = 1 + 3\tan\frac{\pi}{4}$$
$$x = 4$$

we need $\frac{dy}{dx}$.   B1

★ Parametric differentiation:

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ get these by differentiating the parametrics separately.

$y = 2\cos 2t$	$x = 1 + 3\tan t$
$\frac{dy}{dt} = -4\sin 2t$	$\frac{dx}{dt} = 3\sec^2 t$

$$\frac{dy}{dx} = \frac{-4\sin 2t}{3\sec^2 t}$$ M1A1

Hence $\frac{dy}{dx}$ at P:

$$\frac{dy}{dx} = \frac{-4\sin\left(2\cdot\frac{\pi}{4}\right)}{3\sec^2\left(\frac{\pi}{4}\right)}$$

$$\frac{dy}{dx} = -\frac{2}{3}$$

Use $y - y_1 = m(x - x_1)$ to get the equation of the line

$$y - 0 = -\frac{2}{3}(x-2)$$

$$y = -\frac{2}{3}x - 2$$ equation in form $y = mx + c$

dM1A1

(b) Domain is in the x-axis $\therefore$ use parametric equation $x = \ldots$

$x = 1 + 3\tan\left(-\frac{\pi}{6}\right)$ smallest value of t

$= 1 + 3\left(-\frac{\sqrt{3}}{3}\right)$

$\therefore k = 1 - \sqrt{3}$ B1

Question 6 continued

(c) Range is in the y-axis ∴ use parametric equation $y = \dots$

$y_{max} = 2\cos(2 \cdot 0) = 2$

$y_{min} = 2\cos(2 \cdot \frac{\pi}{3}) = -1$ M1

∴ Range: $-1 \leq y \leq 2$ A1

Question 6 continued

(Total for Question 6 is 8 marks)

7. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(i) Use the substitution $u = e^x - 3$ to show that

$$\int_{\ln 5}^{\ln 7} \frac{4e^{3x}}{e^x - 3}\,dx = a + b\ln 2$$

where a and b are constants to be found.

(7)

(ii) Show, by integration, that

$$\int 3e^x \cos 2x \, dx = pe^x \sin 2x + qe^x \cos 2x + c$$

where p and q are constants to be found and c is an arbitrary constant.

(5)

i. (a) To use the given substitution $u = e^x - 3$, we need to find new limits, $e^x = \ldots$ and $dx = \ldots$

New Limits:

x		u
$\ln 7$	$e^{\ln 7} - 3 \rightarrow$	4
$\ln 5$	$e^{\ln 5} - 3 \rightarrow$	2

To get dx, differentiate $u = e^x - 3$

$$\frac{du}{dx} = e^x$$
$$dx = \frac{du}{e^x} = \frac{du}{u+3}$$

Get $e^x = \ldots$

$e^x = u + 3$

B1

Substitute into the Integration:

$$\int_{\ln 5}^{\ln 7} \frac{4e^{3x}}{e^x - 3}\,dx = \int_2^4 \frac{4(u+3)^3}{u} \cdot \frac{du}{u+3}$$

$$= \int_2^4 \frac{4(u+3)^3}{u(u+3)}\,du$$

$$= \int_2^4 \frac{4(u+3)^2}{u}\,du \quad \text{M1A1}$$

$$= \int_2^4 \frac{4(u^2+6u+9)}{u}\,du$$

$$= 4\int_2^4 u + 6 + \frac{9}{u}\,du$$

$$= 4\left[\tfrac{1}{2}u^2 + 6u + 9\ln u\right]_2^4 \quad \text{dM1A1}$$

$$= 4\left(\left(\tfrac{1}{2}(4)^2 + 6(4) + 9\ln 4\right) - \left(\tfrac{1}{2}(2)^2 + 6(2) + 9\ln 2\right)\right) \quad \text{M1}$$

$$= 4\left(8 + 24 + 9\ln 4 - 9\ln 2 - 2 - 12\right)$$

apply ln law $\ln a - \ln b = \ln \frac{a}{b}$

$$= 4(18 + 9\ln 2)$$

$$= 72 + 36\ln 2 \quad \text{A1}$$

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Question 7 continued

ii. $\int 3e^x \cos 2x\, dx$ Multiplication ∴ use By Parts

$\int uv'\, dx = uv - \int vu'\, dx$

$u = e^x \xrightarrow{\frac{d}{dx}} u' = e^x$

$v = \frac{1}{2}\sin 2x \xleftarrow{\int} v' = \cos 2x$

$= 3\left(\frac{e^x}{2}\sin 2x - \int \frac{e^x}{2}\sin 2x\, dx\right)$ By Parts again M1A1

$u = e^x \xrightarrow{\frac{d}{dx}} u' = e^x$

$v = -\frac{1}{2}\cos 2x \xleftarrow{\frac{d}{dx}} v' = \sin 2x$

$= 3\left(\frac{e^x}{2}\sin 2x - \frac{1}{2}\left(-\frac{e^x}{2}\cos 2x + \frac{1}{2}\int e^x \cos 2x\, dx\right)\right)$ dM1

$= 3\left(\frac{e^x}{2}\sin 2x + \frac{e^x}{4}\cos 2x - \frac{1}{4}\int e^x \cos 2x\, dx\right)$ This is our initial integral.

Reintroduce our initial Integral on the LHS and use it to get rid of the infinite "By Parts".

$3\int e^x \cos 2x\, dx = \frac{3e^x}{2}\sin 2x + \frac{3e^x}{4}\cos 2x - \frac{3}{4}\int e^x \cos 2x\, dx$

$\frac{15}{4}\int e^x \cos 2x\, dx = \frac{3}{2}e^x \sin 2x + \frac{3}{4}e^x \cos 2x$ ddM1

$\frac{5}{4}\int 3e^x \cos 2x\, dx = \frac{3}{2}e^x \sin 2x + \frac{3}{4}e^x \cos 2x$

$= \frac{4}{5}\left(\frac{3}{2}e^x \sin 2x + \frac{3}{4}e^x \cos 2x\right)$

$\int 3e^x \cos 2x\, dx = \frac{6}{5}e^x \sin 2x + \frac{3}{5}e^x \cos 2x + c$ A1

Question 7 continued

Question 7 continued

(Total for Question 7 is 12 marks)

8. A student was asked to prove by contradiction that

"there are no positive integers x and y such that $3x^2 + 2xy - y^2 = 25$"

The start of the student's proof is shown in the box below.

Assume that integers x and y exist such that $3x^2 + 2xy - y^2 = 25$

$$\Rightarrow (3x - y)(x + y) = 25$$

If $(3x - y) = 1$ and $(x + y) = 25$

$$\left.\begin{aligned} 3x - y &= 1 \\ x + y &= 25 \end{aligned}\right\} \Rightarrow 4x = 26 \Rightarrow x = 6.5, y = 18.5 \quad \text{Not integers}$$

Show the calculations and statements that are needed to complete the proof.

(4)

There are two more possible cases we need to solve:

I. $(3x-y)=25$ and $(x+y)=1$

$\left.\begin{aligned} 3x-y&=25 \\ x+y&=1 \end{aligned}\right\} \rightarrow 3x-(1-x)=25$

$4x=26 \quad \therefore x=6.5, y=-5.5$

Neither x nor y are integers

M1A1

II. $(3x-y)=5$ and $(x+y)=5$

$\left.\begin{aligned} 3x-y&=5 \\ x+y&=5 \end{aligned}\right\} \quad 3x-(5-x)=5$

$4x=10 \quad \therefore x=2.5, y=2.5$ dM1

Neither x nor y are integers

Hence there are no integers such that $3x^2+2xy-y^2=25$. A1

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Question 8 continued

(Total for Question 8 is 4 marks)

9. With respect to a fixed origin O, the equations of lines l_1 and l_2 are given by

$$l_1 : \mathbf{r} = \begin{pmatrix} 2 \\ 8 \\ 10 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}$$

$$l_2 : \mathbf{r} = \begin{pmatrix} -4 \\ -1 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ 4 \\ 8 \end{pmatrix}$$

where λ and μ are scalar parameters.

Prove that lines l_1 and l_2 are skew.

(5)

Try to equate the general points of the lines.

gen. point of l_1 $\begin{pmatrix} 2-\lambda \\ 8+2\lambda \\ 10+3\lambda \end{pmatrix} = \begin{pmatrix} -4+5\mu \\ -1+4\mu \\ 2+8\mu \end{pmatrix}$ gen. point of l_2

Let's equate components of x and y and do simultaneous equations to get values for λ and μ.

x $\quad 2-\lambda = -4+5\mu \quad |\times 2| \quad 4-\cancel{2\lambda} = -8+10\mu$

y $\quad 8+2\lambda = -1+4\mu \quad \Rightarrow \quad 8+\cancel{2\lambda} = -1+4\mu \quad +$ use elimination

$12 = -9+14\mu$

$21 = 14\mu \Rightarrow \mu = \frac{3}{2}$.

$2-\lambda = -4+5\left(\frac{3}{2}\right)$ solve for λ

$\lambda = -\frac{3}{2}$ M1A1

Now substitute our $\mu = \frac{3}{2}$ and $\lambda = -\frac{3}{2}$ into z to try if the values satisfy the equation:

$10+3\left(-\frac{3}{2}\right) = 2+8\left(\frac{3}{2}\right)$

$\hookrightarrow \frac{11}{2} \neq 14$ dM1

Hence these values do not satisfy the equation, hence they do not represent a point of intersection.

Additionally, the direction vectors, $\begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 5 \\ 4 \\ 8 \end{pmatrix}$, are not parallel.

So, the lines are skew. A1A1

Question 9 continued

(Total for Question 9 is 5 marks)

10. A spherical ball of ice of radius 12 cm is placed in a bucket of water.

In a model of the situation,

- the ball remains spherical as it melts
- t minutes after the ball of ice is placed in the bucket, its radius is r cm
- the rate of decrease of the radius of the ball of ice is inversely proportional to the square of the radius
- the radius of the ball of ice is 6 cm after 15 minutes

Using the model and the information given,

(a) find an equation linking r and t, **(5)**

(b) find the time taken for the ball of ice to melt completely. **(2)**

(c) On Diagram 1 on page 27, sketch a graph of r against t. **(1)**

(a) "rate of decrease of the radius of the ball of ice is inversely proportional to the square of the radius"

Hence: $\frac{dr}{dt} \propto \pm\frac{1}{r^2}$. B1

$\frac{dr}{dt} = -\frac{k}{r^2}$ Solve this differential equation

$\int r^2\,dr = \int -k\,dt$ Separate variables by grouping the like terms on one side

$\frac{1}{3}r^3 = -kt + c$ Integrate

Use the given $t=0, r=12$ to get c: M1A1

$\frac{1}{3}(12)^3 = -k(0) + c$

$\therefore c = 576$

Substitute $t=15, r=6$ to get k:

$\frac{1}{3}(6)^3 = -k(15) + 576$

$-504 = -15k$

$k = 33.6$ M1

Hence the equation: $\frac{1}{3}r^3 = 576 - 33.6t$ A1

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Question 10 continued

(b) We want t when $r=0$:

$$\frac{1}{3}(0)^3 = 576 - 33.6t$$ solve for t

M1

$$33.6t = 576$$

$$\boxed{t = 17.1 \text{ min}}$$ A1

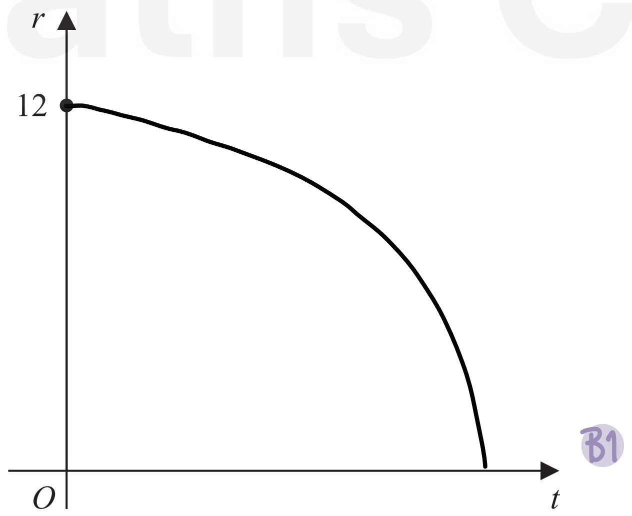

Diagram 1

P 7 2 0 6 8 A 0 2 7 3 2

Question 10 continued

Question 10 continued

(Total for Question 10 is 8 marks)

11.

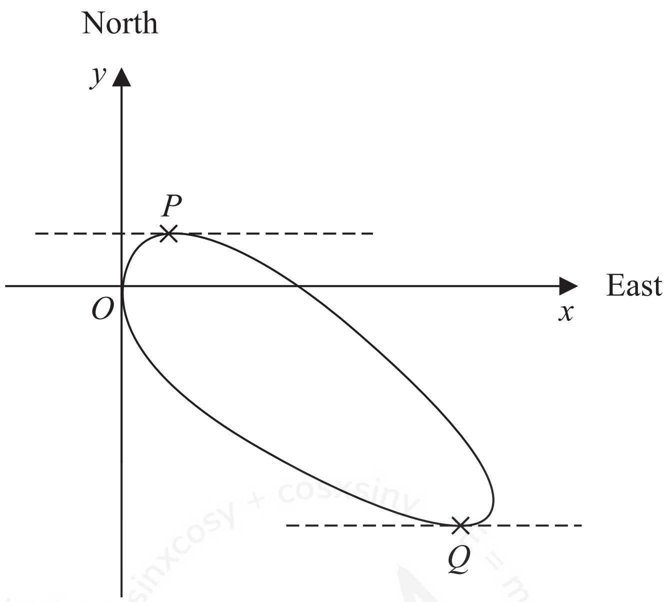

Figure 4

Figure 4 shows a sketch of the closed curve with equation

$$(x+y)^3 + 10y^2 = 108x$$

(a) Show that

$$\frac{dy}{dx} = \frac{108 - 3(x+y)^2}{20y + 3(x+y)^2}$$

(5)

The curve is used to model the shape of a cycle track with both x and y measured in km.

The points P and Q represent points that are furthest north and furthest south of the origin O, as shown in Figure 4.

Using the result given in part (a),

(b) find how far the point Q is south of O. Give your answer to the nearest 100 m.

(4)

Question 11 continued

(a)

★ Implicit differentiation:

When differentiating y, multiply by $\frac{dy}{dx}$

$$(x+y)^3 + 10y^2 = 108x$$

$$3(x+y)^2 \cdot \left(1+\frac{dy}{dx}\right) + 20y\frac{dy}{dx} = 108 \quad \text{B1 M1 A1}$$

chain rule: multiply by the derivative of the bracket

$$3(x+y)^2 + 3(x+y)^2\frac{dy}{dx} + 20y\frac{dy}{dx} = 108$$

$$3(x+y)^2\frac{dy}{dx} + 20y\frac{dy}{dx} = 108 - 3(x+y)^2$$ collect all $\frac{dy}{dx}$ on one side dM1

$$\frac{dy}{dx}\left(3(x+y)^2 + 20y\right) = 108 - 3(x+y)^2$$ factor $\frac{dy}{dx}$ out

$$\frac{dy}{dx} = \frac{108 - 3(x+y)^2}{3(x+y)^2 + 20y}$$ hence shown A1

(b) Points P and Q occur when $\frac{dy}{dx} = 0$.

$$0 = \frac{109 - 3(x+y)^2}{3(x+y)^2 + 20y}$$

$$0 = 108 - 3(x+y)^2$$

$$(x+y)^2 = 36 \quad \text{M1}$$

$$(x+y) = \pm 6$$

$$\Rightarrow x = 6 - y$$

Substitute $x = 6-y$ back into $(x+y)^3 + 10y^2 = 108x$ and solve for y:

$$(6-y+y)^3 + 10y^2 = 108(6-y) \quad \text{dM1}$$

$$216 + 10y^2 = 648 - 108y$$

$$10y^2 + 108y - 432 = 0$$

Use Quadratic Formula: $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$$y = \frac{-108 \pm \sqrt{108^2 - 4(10)(-432)}}{20}$$

north point $y = 3.106$ $y = -13.906$ south point. ddM1

$\therefore Q_y = 13900\,\text{m}$ A1

Question 11 continued

(Total for Question 11 is 9 marks)

TOTAL FOR PAPER IS 75 MARKS